

# MATHEMATICS COLLOQUIUM

## *Some General Non-linear Variational and Variational-like Inequalities*

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**Abstract:** The subject of variational inequalities (VIs) has its origin in the calculus of variations associated with the minimization of infinite dimensional functionals. The systematic study of the subject began in the early 1960s with the seminal work of the Italian mathematician Guido Stampacchia and his collaborators, who used the variational inequalities as an analytic tool for studying free boundary value problems defined by non-linear partial differential operators arising from problems in elasticity, plasticity theory and in mechanics. Many interesting questions in the theory of variational inequalities may be formulated in the terms of bilinear forms on Hilbert spaces. This theory is a generalization of the variational theory of boundary value problems for linear elliptic equations. The theorems of the existence and uniqueness were proved by Stampacchia and Lions and Stampacchia using fixed point techniques in both cases. In the study of variational inequalities in general spaces, the property of monotonicity becomes important for the existence of a solution. There are close connections between convex functions and monotone operators. There were some interesting general nonlinear variational inequalities in Hilbert space setting. Bose considered some of these variational inequalities. The existence and uniqueness of solutions were established by two fixed point techniques. It was concluded that the approach of Glowinski, Lions and Tremolieres gives better results compared to fixed point techniques of Lions and Stampacchia. The minimax inequality of Ky Fan is very crucial in proving many existence theorems in non-linear analysis, in particular, in variational inequality problems. There have been many generalizations of Ky Fan's inequality by weakening the compactness assumptions or the convexity assumptions (like  $\gamma$ -diagonally concavity). Bose proved some general VIs in locally convex topological vector space setting using generalized convexity and relaxing compactness and using a generalization of Ky Fan's inequality by Zhan and Chen. Recently the results of Bose in Hilbert space setting have been extended to reflexive Banach space setting where generalized Ky Fan's inequality has been used and the mappings considered are more general. Also recently two classes of variational-like inequalities concerning generalized monotone set-valued mappings ( $\eta - \alpha$  relaxed monotone) in reflexive Banach space setting are presented. The existence of a solution was proved by using KKK mapping and KKM-Fan theorem.

### **Selected References**

R. K. Bose, Variational-like inequalities for generalized set-valued mapping in reflexive Banach spaces, Bulletin of Pure and Applied Mathematics(2007)

R. K. Bose, On a general non-linear variational inequality in reflexive Banach spaces, Pacific Journal of Mathematics( 2007)

J. Lions and G. Stampacchia, Variational Inequalities, Comment Pure Appl. Math 20(1967), 493-519

R. K. Bose, Ky Fans Inequality and Nonlinear Variational Inequalities, Nonlinear Analysis, Methods and Applications 30(1997) 4161-4170

R. K. Bose, On a General Nonlinear Variational Inequalities, Bull Austral Math Soc.,42(1990) 399-406

Date:           Tuesday, **February 12, 2008**  
Time:           4:00 pm – 5:00 pm  
Place:          J. Wiener Lecture Hall, MAGC 1.302

Refreshments will be served at 3:50pm.

For further information or for special accommodations, contact Dr. Karen Yagdjian at 381-2145, via email at [yagdjian@utpa.edu](mailto:yagdjian@utpa.edu), or visit [www.math.panam.edu/colloquia.html](http://www.math.panam.edu/colloquia.html)