

MATHEMATICS COLLOQUIUM

Quantization dimension function for ergodic Markov measure

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Abstract: For over fifty years electrical engineers and mathematicians have been interested in the problem of efficiently “quantizing” a probability distribution in the sense of estimating a given probability by a discrete probability supported on a finite set. This problem arises in signal processing, data compression, cluster analysis, and pattern recognition, and it also has been studied in the context of economics, statistics, and numerical integration. Many useful theorems and algorithms have evolved over the years. Two main goals have been (1) finding the exact configuration of a so-called “ n -optimal set” which corresponds to the support of the quantized version of the distribution, and (2) estimating the rate (called “quantization dimension”) at which some specified measure of the error (also called the distortion or noise, between the quantized distribution and the original distribution) goes to zero as n goes to infinity. Given a Borel probability measure μ on \mathbb{R}^d , a number $r \in (0, +\infty)$ and a natural number $n \in \mathbb{N}$, the n -th *quantization error* of order r for μ , is defined by

$$e_{n,r} = \inf\left\{\left(\int d(x, \alpha)^r d\mu(x)\right)^{\frac{1}{r}} : \alpha \subset \mathbb{R}^d, \text{card}(\alpha) \leq n\right\},$$

where $d(x, \alpha)$ denotes the distance from the point x to the set α with respect to a given norm $\|\cdot\|$ on \mathbb{R}^d . If $\int \|x\|^r d\mu(x) < \infty$ then there is some set α for which the infimum is achieved. This set α can then be used to give a best approximation of μ by a discrete probability supported on a set with no more than n points. The quantization dimension of order r for μ is defined to be

$$D_r = D_r(\mu) = \lim_{n \rightarrow \infty} \frac{\log n}{-\log e_{n,r}},$$

if the limit exists. If the limit does not exist then we define \overline{D}_r as the lim sup of the sequence and \underline{D}_r as the lim inf. In this paper quantization dimension function for the image measure of ergodic Markov measure on self similar fractals has been constructed, and a relation between the quantization dimension function and the thermodynamic formalism has been established.

Date: Friday, **March 17, 2008**
Time: 4:00 pm – 5:00 pm
Place: J. Wiener Lecture Hall, MAGC 1.302

Refreshments will be served at 3:50pm.

For further information or for special accommodations, contact Dr. Karen Yagdjian at 381-2145, via email at yagdjian@utpa.edu, or visit www.math.panam.edu/colloquia.html