

Mathematical Modeling of Bio–electromagnetic Information Processing the Extracellular Potentials and Fields of a Traveling Gaussian Wave Approximation of the Action Potential for a Neural Volume Conductor

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The action potential is the primary means of intercellular communications in the neuron. It is a self–initiating, self–propagating, self–limiting, and self–restoring phenomenon that causes a uniform electrical impulse to travel along a nerve fiber. The action potential has been extensively studied, and it is fairly well understood. Several mathematical models predict the behavior of this pulse from the transport of ions across the nerve membrane. In these models, the electrical pulse is the solution to a set of ordinary differential equations (as in the Hodgkin–Huxley model) or a set of partial differential equations (as in the Fitzhugh–Nagumo model).

This work proposes the assumption of a traveling pulse that is qualitatively and quantitatively similar to the pulse produced by the action potential. Instead of deriving the shape and propagation characteristics of the action potential from solving a system of differential equations, the traveling Gaussian wave is assumed to approximate the behavior of the action potential. The Gaussian wave introduces voltage and timing scaling factors that make this wave a good model for the behavior of the action potential.

The choice of the Gaussian wave arises from both its similarity to the action potential and the fact that it is well–behaved under differentiation and integration. From the assumed Gaussian pulse, equations are derived for the electric field, the magnetic field, the scalar potential, and the vector potential for any point exterior to the nerve fiber. These field equations for the single volume conductor case will provide the foundation for future research in modeling systems of multiple active conductors.