On the Hamiltonian Constraint Equation in General Relativity

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In general relativity, spacetime is a 4–dimensional manifold of events endowed with a pseudo–Riemannian metric. This metric determines curvature on the manifold, and Einstein's equations relate the curvature at a point of spacetime to the mass–energy there. Einstein's equations can be viewed as equations for geometries, that is, their solutions are equivalent classes under spacetime diffeomorphisms of metric tensors. To break this diffeomorphism invariance, Einstein's equations must be first transformed into a system having a well–posed Cauchy problem. In other words, the spacetime is foliated and each slice is characterized by its intrinsic geometry and extrinsic curvature. This procedure allows one to express Einstein's equations as a system of evolution equations together with a set of constraints (the momentum and Hamiltonian constraints). The problem of determining a solution to the differential system formed by the constraints is known as the initial data problem. Based on an older idea originated by Lichnerowicz, York and Piran describe a general method of converting the constraints into a system of four semilinear elliptic equations whose solution provide a foundation for prescribing the Cauchy data on the initial slice. The base of the method consists in specifying the physical data up to conformal equivalence. In this talk, we discuss the existence and uniqueness of the conformal factor for the Hamiltonian constraint in the case of a single black–hole. This is a joint work with Prof. Douglas Arnold (IMA–University of Minnesota).