Project #4 – Math 3368: Numerical Methods – Spring 2009

Due day: April 30, 2009

1. Suppose that an object can be at any one of $n+1$ equally spaced points $x_0, x_1, \cdots, x_n$. When an object is at location x_i, it is equally likely to move to either x_{i-1} or x_{i+1} and cannot directly move to any other location. Consider the probabilities P_i that an object starting at location x_i will reach the left endpoint x_0 before reaching the right endpoint x_n. Clearly, $P_0 = 1$, $P_n = 0$. Since the object can move to x_i only from x_{i-1} or x_{i+1} and does so with probability $\frac{1}{2}$ for each of these locations, i.e., $P_i = \frac{1}{2}P_{i-1} + \frac{1}{2}P_{i+1}$, $i = 1, 2, \cdots, n-1$.

- Show that

$$\begin{bmatrix} 1 & -\frac{1}{2} & 0 & & & 0 \\ -\frac{1}{2} & 1 & -\frac{1}{2} & \ddots & & \\ 0 & -\frac{1}{2} & 1 & \ddots & \ddots & \\ & \ddots & \ddots & \ddots & \ddots & 0 \\ & & \ddots & \ddots & \ddots & -\frac{1}{2} \\ 0 & & & 0 & -\frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ \vdots \\ P_{n-2} \\ P_{n-1} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ 0 \\ \vdots \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

- Write a program to solve a linear system $Ax = b$ by using the method of LU decomposition. Solve this system using $n = 10, 50, 100$.

2. Write programs by using the Jacobi and Gauss-Seidel iterative methods. Solve the above linear system in problem 1 with $TOL = 10^{-6}$ in the l_2 norm.